Total No. of Questions : 9]

SEAT No. :

PD-4045

[Total No. of Pages : 5

[6402]-4

S.E. (Civil)

ENGINEERING MATHEMATICS -III

(2019 Pattern) (Semester-III) (207001)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Question no. 1 is compulsory.*
2) *Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7 and Q.8 or Q.9.*
3) *Assume Suitable data if necessary.*
4) *Neat diagrams must be drawn whenever necessary.*
5) *Figures to the right indicate full marks.*
6) *Use of electronic pocket Calculator is allowed.*

Q1) a) If $f(x)=\frac{1}{\sqrt{6\pi}}e^{-x^2/6}, x\in(-\infty,\infty)$ then the mean μ and variance σ^2 are **[1]**

i) $\mu = 1, \sigma^2 = 3$

ii) $\mu = 0, \sigma^2 = 3$

iii) $\mu = 0, \sigma^2 = \sqrt{3}$

iv) $\mu = 3, \sigma^2 = 1$

b) The regression lines y on x is given by $2x - 3y + 5 = 0$. The slope b_{xy} of the regression line x on y satisfies. **[2]**

i) $b_{xy} = 1$

ii) $b_{xy} < 3/2$

iii) $b_{xy}\, b_{yx} = 1$

iv) $b_{xy}\, b_{yx} > 1$

P.T.O.

c) If the vector field $\bar{F}=m(x+z)\bar{i}-2(y+z)\bar{j}-z\bar{k}$ is solenoidal then value of m is **[2]**

i) -3 ii) 3

iii) 4 iv) 0

d) Unit vector along the direction of line $\frac{x}{-1}=\frac{y-2}{3}=\frac{z+1}{2}$ is **[1]**

i) $\frac{\bar{i}+\bar{j}+\bar{k}}{\sqrt{3}}$ ii) $\frac{\bar{i}+3\bar{j}+2\bar{k}}{\sqrt{14}}$

iii) $\frac{-\bar{i}+3\bar{j}+2\bar{k}}{\sqrt{14}}$ iv) $\frac{-\bar{i}+3\bar{j}+\bar{k}}{\sqrt{11}}$

e) The value of $\iiint_V \nabla\cdot\bar{F}\,dv$ where $\bar{F}=yz\bar{i}+xz\bar{j}+xy\bar{k}$ over the surface of sphere is **[2]**

i) 3 ii) 0

iii) 4 iv) 10

f) The most general solution of $\frac{\partial u}{\partial t}=9\frac{\partial^2 u}{\partial x^2}$ is **[2]**

i) $u(xt) = (C_4\cos mx + C_5 \sin mx)\, e^{-9m^2t}$

ii) $u(x,t) = (C_1\cos mx + C_2 \sin mx)\, e^{-3m^2t}$

iii) $u(xt) = (C_1e^{mx} + C_2 e^{-mx})\left(C_3 \cos mx + C_4 \sin mx\right)$

iv) $u(xt) = (C_1\cos mx + C_2 \sin mx)\left(C_3 \cos ct + C_4 \sin ct\right)$

Q2) a) The first three moments $\mu_1^1, \mu_2^1, \mu_3^1$ about the value 2 of a distribution are 2, 32 and -80. Find the mean, standard deviation and the coefficient of skewness of the distribution. **[5]**

b) A set of 10 coins are tossed 4096 times. Assuming that the coins are identical and fair, in how many cases do you expect: **[5]**

i) 8 heads and 2 tails?

ii) at least 8 heads?

c) Fit a Poisson distribution to the following data and find the χ^2 value.

x	0	1	2	3	4
f	122	60	15	2	1

Here f denotes frequency. Take $e^{\frac{-1}{2}} = \frac{3}{5}$. Round off frequencies to the immediate higher integer values. **[5]**

OR

Q3) a) 10% rivets produced by a machine are defective. Find the probability that out of 10 rivets chosen at random.

i) none will be defective.

ii) one will be defective

iii) at least one will be defective.

Apply the Poisson random variable theory to solve this question. **[5]**

b) In every 30 days rain falls on 10 days on an average. Obtain the probability that **[5]**

i) rain will fall on at least 3 days of a week.

ii) the first 3 days of a week will be dry and the remaining 4 days wet.

c) The monthly wages of 10,000 workers in a factory follows normal distribution with mean and standard deviation as ₹17000 and ₹1000 respectively. Find the expected number of workers whose monthly wages are between ₹16000 and ₹20,000. Take area $(0 < z < 1) = 0.34$ and area $(0 < z < 3) = 0.49$ where z is the standard normal variate. **[5]**

Q4) a) Prove the following identities (any one) **[5]**

i) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$

ii) $\nabla^4 \left(r^2 \log r\right) = \frac{6}{r^2}$

b) Find Directional derivative of $\phi = xy^2 + yz^2$ at $(2,-1,1)$ along the line $2(x-2) = y+1 = z-1$. **[5]**

c) Show that $\vec{F}=\left(2xz^3+6y\right)\hat{i}+\left(6x-2yz\right)\hat{j}+\left(3x^2z^2-y^2\right)\hat{k}$ is irrotational. Find ϕ such that $\vec{F}=\nabla\phi$. **[5]**

OR

Q5) a) For a solenoidal vector field $\vec{F}$, show that curl curl curl curl $\vec{F}=\nabla^4\vec{F}$ **[5]**

b) Find the directional derivative of $\phi=x^2-y^2-2z^2$ at the point P(2,–1,3) in the direction PQ where Q is (5,6,4). **[5]**

c) Show that $\vec{F}=\left(y^2\cos x+z^2\right)\hat{i}+\left(2y\sin x\right)\hat{j}+2xz\hat{k}$ is irrotational Find ϕ such that $\vec{F}=\nabla\phi$. **[5]**

Q6) a) Evaluate $\int_C \overline{F}.d\overline{r}$ where $\overline{F}=x^2\overline{i}+xy\overline{j}$ and C is straight line $y = x$ joining points (0,0) and (1,1). **[5]**

b) By using Gauss divergence theorem evaluate $\iint_S \overline{F}.d\overline{s}$ for vector field $\overline{F}=yz\,\overline{i}+zx\,\overline{j}+xy\overline{k}$ where S is curved surface of cone $x^2+y^2 = z^2$, $z = 4$. **[5]**

c) Use Stoke's theorem to evaluate $\int_C \overline{F}.d\overline{r}$ where $\overline{F}=e^x\,\overline{i}+2y\,\overline{j}-\overline{k}$ where 'C' is curve $x^2+y^2 = 4$, $z = 2$. **[5]**

OR

Q7) a) Find work done in moving a particle around ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ in plane z = 0 where $\overline{F}=\left(3x-2y\right)\overline{i}+\left(2x+8y\right)\overline{j}+y^2\overline{k}$. **[5]**

b) Use Gauss divergence theorem to show that $\iint_S \frac{\overline{r}}{r^2}.d\overline{s}=\iiint_V \frac{dv}{r^2}$ **[5]**

c) Evaluate $\iint_S (\nabla \times \bar{F}).d\bar{S}$ where $\bar{F} = (x^2 + y - 4)\bar{i} + 3xy\,\bar{j} + (2xz + z^2)\bar{k}$ over the surface of hemisphere $x^2+y^2+z^2 = 16$ above XOX plane. **[5]**

Q8) a) A taut string of length $2l$ is fastened at both ends, the midpoint of the string is taken to the hight 'b' and the released from the rest in this position. Obtain the displacement $y(x,t)$ if $\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$. **[8]**

b) Solve the one diemensional heat equation, $\frac{\partial u}{\partial t} = K \frac{\partial^2 y}{\partial x^2}$, subject to conditions. **[7]**

i) u is finite $\forall t$

ii) $u\ (0,t) = 0$

iii) $u\ (100,t) = 0$,

iv) $u(x,0) = \begin{cases} x & if \quad 0 \le x \le 50 \\ 100 - x & if \quad 50 \le x \le 100 \end{cases}$

OR

Q9) a) If a string of length 4cm is initially at rest in its equilibrium position is set to vibration by giving each point a velocity,

$$\frac{\partial y}{\partial t}\Big|_{t=0} = \begin{cases} 3x & if \quad 0 \le x \le 2 \\ 3(4-x) & if \quad 2 \le x \le 4 \end{cases}$$

find the displacement $y(x,t)$

[8]

b) An infinitely long uniform metal plate is enclosed between two parallel edges $x = 0$ and $x = \pi$ For y>0. The temperature is zero along the edges $x = 0$, $x = \pi$ and at infinity. If edge $y = 0$ is kept at a constant temperatute V_0. Find the temperature distribution. V (x,y). **[7]**

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