Total No. of Questions : 9]

SEAT No. :

PC2785

[Total No. of Pages : 5

[6352]-9

S.E. (Civil)

ENGINEERING MATHEMATICS - III

(2019 Pattern) (Semester - III) (207001)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question No. 1 is compulsory.*
2) *Answer Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
3) *Non-programmable electronic pocket calculator is allowed.*
4) *Figures to the right indicate full marks.*
5) *Assume suitable data if necessary.*
6) *Neat diagrams must be drawn wherever necessary.*

Q1) Write the correct option for the following multiple choice questions :

a) Given $b_{xy} = 0.85$, $b_{yx} = 0.89$ and $\sigma_x = 6$ then the values of $r(x,y)$ and σ_y are **[2]**

i) $r = 0.87, \sigma_y = 6.14$

ii) $r = -0.87, \sigma_y = 0.61$

iii) $r = 0.75, \sigma_y = 6.14$

iv) $r = 0.89, \sigma_y = 4.64$

b) The divergence of a vector field $\overline{F} = 3xz\overline{i} + 2xy\overline{j} - yz^2\overline{k}$ at a point (1, 1, 1) is **[2]**

i) 4

ii) 6

iii) 5

iv) 3

P.T.O.

c) If $\overline{F} = y^2\overline{i} + z^2\overline{j} + x^2\overline{k}$, then by using divergence theorem, $\iint_s \overline{F}.d\overline{s} =$

(where S is closed surface bounded by volume V) **[2]**

i) 3

ii) 0

iii) 2

iv) 5

d) The finite and bounded general solution of heat equation $\frac{\partial u}{\partial t} = 5\frac{\partial^2 u}{\partial x^2}$ is **[2]**

i) $u(x,t) = (c_1 \cos mx + c_2 \sin mx)(c_3 \cos 5\text{mt} + c_4 \sin 5\text{mt})$

ii) $u(x,t) = (c_1 \cos mx + c_2 \sin mx)e^{-25mt}$

iii) $u(x,t) = (c_1 \cos mx + c_2 \sin mx)e^{-5mt}$

iv) $u(x,t) = (c_1 e^{-mt} + c_2 e^{-mx})(c_3 \cos mx + c_4 \sin mn)$

e) If x is a poisson random variable with mean value 3 then standard deviation of poisson distribution is **[1]**

i) 1

ii) $\sqrt{2}$

iii) $\sqrt{3}$

iv) 3

f) $\nabla \times \overline{r} =$ **[1]**

i) 0

ii) 1

iii) 2

iv) 3

Q2) a) First four moments of the distribution are 1, 4, 10 and 46. Compute the first four central moments. Also find β_1 and β_2. **[5]**

b) Find the Co-efficient of correlation for the following data. **[5]**

x	152	158	169	182	160	166	182
y	198	178	167	152	180	170	162

c) With the usual notations, find the probability of the binomial distribution (p) if 9 P(X = 4) = P(X = 2) where X be the random variable. **[5]**

OR

Q3) a) If $\overline{x}=8.2, \overline{y}=12.4, \sigma_x=6.2\ \sigma_y=20, r(x,y)=0.9$ then find the line of regression y on x. Also kind the value of y for x = 10. **[5]**

b) If the random variable X follows the poisson distribution such that P(X = 1) = 2P (X = 2), **[5]**

find the

i) mean of distribution

ii) P(X = 3)

c) From the past experience the labor contractor knows that per hour wages of skilled labor on an average is Rs. 100 with standard deviation Rs. 2. What percentage of labors will have wages between Rs. 98 and Rs. 102, assuming that the wages are normally distributed . **[5]**

(Given : $\phi(1) = \phi(-1) = 0.3413$)

Q4) a) Find the angle between velocity and acceleration vectors to the curve $x=2\sin(3t), y=2\cos(3t), z=8t$ at t = 0. **[5]**

b) Find the directional derivative of the function $\phi(x, y, z) = 4\ e^{2x-y-z}$ at (1, 1, 1) in the direction tangent to the curve $x = e^{-t}\cos t$ $y = 2\sin t + 1$, $z = t - \cos t$ at $t = 0$. **[5]**

c) Show that vector field **[5]**

$$\overline{F}=(x+2y+4z)\overline{i}+(2x-3y-z)\overline{j}+(4x-y+2z)\overline{k}$$

is irrotational. Find scalar potential ϕ such that $\overline{F}=\nabla\phi$.

OR

Q5) a) Find the angle between surfaces $x^2 - y^2 + 2z^2 = 3$ and $x^2 + y^2 + z^2 = 16$ at (1, 2, 2). **[5]**

b) Determine $f(r)$, such that $\overline{F} = f(r).\overline{r}$ is solenoidal. **[5]**

c) Attempt any one. **[5]**

i) Prove that : $\nabla^2\left(\frac{1}{r^2}\right) = \frac{2}{r^4}$

ii) Prove that : $\nabla^2(\phi\psi) = \phi\nabla^2\psi + 2\nabla\phi\nabla\psi + \psi\nabla^2\phi$

Q6) a) Evaluate $\int_C \overline{F}\cdot d\overline{r}$ for $\overline{F} = 3x^2\overline{i} + (2xz - y)\overline{j} + z\overline{k}$ along the curve $x = 2t^2, y = t, z = 4t^2 - t$ from $t = 0, t = 1$. **[5]**

b) Evaluate $\iint_S (\nabla \times \overline{F})\cdot d\overline{s}$ where $\overline{F} = 3y\overline{i} - xz\overline{j} + yz^2\overline{k}$ and S is the surface of the paraboloid $x^2 + y^2 = 2z$ bounded by the plans $z = 2$. **[5]**

c) Evaluate $\iint_S \overline{F}\cdot d\overline{s}$ by using Gauss-divergence theorem, where $\overline{F} = (4x + 3yz^2)\overline{i} - (x^2z^2 + y)\overline{j} + (y^3 + 2z)\overline{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 9$. **[5]**

OR

Q7) a) Using Green's theorem, evaluate $\int_C \overline{F}\cdot d\overline{r}$, wher $\overline{F} = (\sin y)\overline{i} + x(1 + \cos y)\overline{j}$ and C is the boundary of an ellipse $\frac{x^2}{4} + \frac{y^2}{g} = 1; z = 0$. **[5]**

b) Using Stoke's theorem, evaluate $\int_C \overline{F}\cdot d\overline{r}$, where $\overline{F} = yz\overline{i} + zx\overline{j} + xy\overline{k}$ and S is the upper part of the sphere $x^2 + y^2 + z^2 = 1$ above XOY plane. **[5]**

c) Evaluate $\iint_S (x^3\overline{i} + y^3\overline{j} + z^3\overline{k})\cdot d\overline{s}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **[5]**

Q8) a) Solve one dimensional heat flow equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ subject to the conditions. **[7]**

i) $u(0,t)=0$

ii) $u(\pi,t)=0$

iii) $u(x,0)=\pi x-x^2, 0<x<\pi$

b) Solve wave equation $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$ subject to conditions **[8]**

i) $y(0,t)=0$

ii) $y(L,t)=0$

iii) $\left.\frac{\partial y}{\partial t}\right|_{t=0}=0$

iv) $y(x,0)=y_0; 0<x<L$

where y_0 is constant

OR

Q9) a) Solve Laplace equation $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=0$ subject to conditions : **[7]**

i) $v(0,y)=0$

ii) $v(1,y)=0$

iii) $v(x,\infty)=0, 0<x<1$

iv) $v(x,0)=10, 0<x<1$

b) Solve wave equation $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$ subject to conditions **[8]**

i) $y(0,t)=0$

ii) $y(\pi,t)=0$

iii) $\left.\frac{\partial y}{\partial t}\right|_{t=0}=0$

iv) $y(x,0)=2\sin x, 0<x<\pi$

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