Total No. of Questions—8] [Total No. of Printed Pages—7

Seat No.	

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S.E. (Civil) (I Sem.) EXAMINATION, 2018

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Draw neat diagrams wherever needed.

(*iii*) Figures to the right indicate full marks.

(*iv*) Assume suitable data, if necessary.

(*v*) Use of non-programmable pocket calculator is allowed.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $\left(D^3 - D^2 + 4D - 4\right) y = e^x$

(*ii*) $\left(D^2 - 2D + 2\right) y = e^x \tan x$

(by method of variation of parameters)

(*iii*) $x^3 \frac{d^3 y}{dx^3} + x^2 \frac{d^2 y}{dx^2} - 2y = \frac{1}{x^3}$.

(*b*) Apply Gauss-Jordan method to solve the equations : [4]

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40.$$

P.T.O.

Or

2. (*a*) A light horizontal strut AB of length l is freely pinned at A & B and is under the action of equal and opposite compressive forces P at each of its ends and carries a load W at its centre. Show that the deflection at its centre is : [4]

$$\frac{W}{2P}\left[\frac{1}{n}\tan\frac{nl}{2} - \frac{l}{2}\right] \text{ where } n^2 = \frac{P}{EI}.$$

(*b*) Using fourth order Runge-Kutta method solve the equation : [4]

$$\frac{dy}{dx} = \sqrt{x + y}$$

subject to the conditions $x = 0$, $y = 1$ to find y at $x = 0.1$ taking $h = 0.1$.

(*c*) Solve the following system by Cholesky's method : [4]

$$9x_1 + 6x_2 + 12x_3 = 17.4$$
$$6x_1 + 13x_2 + 11x_3 = 23.6$$
$$12x_1 + 11x_2 + 26x_3 = 30.8.$$

3. (*a*) The equation of two lines of regression obtained in a correlation analysis are the following : [4]

$$2x + 3y - 8 = 0 \text{ and}$$
$$x + 2y - 5 = 0.$$

Obtain the value of the correlation co-efficient and the variance of y given that variance of x is 12.

(*b*) An aptitude test for selecting officiers in a bank conducted on 1000 candidates. The average score is 42 and standard deviation of score is 24. Assuming normal distribution for the score find :

(*i*) The number of candidates whose scores exceed 60.

(*ii*) The number of candidates whose score lie between 30 and 60.

[Given Area = 0.2734 for z = 0.75, Area = 0.1915 for z = 0.5.]

(*c*) Find the directional derivative of $\phi = 4e^{2x - y + z}$ at the point (1, 1, –1) in the direction towards (–3, 5, 6). [4]

Or

4\. (*a*) In a certain distribution the first four moments about 4, are 1.5, 17, 30 and 108. Find the central moments and hence β_1 and β_2. [4]

(*b*) Prove the following (any *one*) : [4]

(*i*) $\nabla\left(\frac{\bar{a}\cdot\bar{r}}{r^n}\right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a}\cdot\bar{r})}{r^{n+2}}\,\bar{r}$

(*ii*) $\bar{a}\cdot\nabla\left[\bar{b}\cdot\nabla\left(\frac{1}{r}\right)\right] = \frac{3(\bar{a}\cdot\bar{r})(\bar{b}\cdot\bar{r})}{r^5} - \frac{\bar{a}\cdot\bar{b}}{r^3}$.

(*c*) Show that the vector field : [4]

$$\bar{F} = \left(y^2\cos x + z^2\right)\hat{i} + (2y\sin x)\,\hat{j} + 2xz\,\hat{k}$$

is conservative and find scalar field such that $\bar{F} = \nabla\phi$.

5. Attempt any *two* :

(*a*) Using Green's theorem evaluate : [6]

$$\oint \bar{F} \cdot d\bar{r}$$

for the field

$$\bar{F} = 2x^2 y \,\bar{i} + x^3 \,\bar{j}$$

over the first quadrant of the circle $x^2 + y^2 = a^2$.

(*b*) Using Divergence theorem evaluate : [6]

$$\iint_S \left(y^2 z^2 \,\bar{i} + z^2 x^2 \,\bar{j} + x^2 y^2 \,\bar{k}\right) \cdot d\bar{S},$$

where S is the upper half of sphere $x^2 + y^2 + z^2 = 4$ above the plane $z = 0$.

(*c*) Evaluate : [7]

$$\iint_S \nabla \times \bar{F} \cdot \hat{n} \, dS,$$

where

$$\bar{F} = (x - y)\,\bar{i} + \left(x^2 + yz\right)\bar{j} - 3xy^2 \,\bar{k}$$

and S is the surface of the cone $z = 4 - \sqrt{x^2 + y^2}$ above the XOY-plane.

Or

6. Attempt any *two* :

(*a*) Find the work done in moving a particle along $x = 3\cos\theta$, $y = 3\sin\theta$, $z = 5\theta$, from $\theta = \frac{\pi}{4}$ to $\theta = \frac{\pi}{2}$ under a field of force given by : [6]

$$\bar{F} = -9\sin^2\theta\cos\theta \,\bar{i} + 3\left(2\sin\theta - 3\sin^3\theta\right)\bar{j} + 5\sin 2\theta \,\bar{k}.$$

(*b*) Evaluate : [6]

$$\iint_S \left(x\,\bar{i} + y\,\bar{j} + z\,\bar{k}\right) \cdot d\bar{S}$$

where S is the curved surface of the cylinder $x^2 + y^2 = 4$ bounded by the planes $z = 0$ and $z = 2$.

(*c*) Evaluate : [7]

$$\iint_S \left(\nabla \times \bar{F}\right) \cdot d\bar{S}$$

where

$$\bar{F} = x^3\,\bar{i} - xyz\,\bar{j} + y^3\,\bar{k}$$

and S is the surface $x^2 + 9y^2 + 4z^2 - 2x = 36$ above the plane $x = 0$.

7. Solve any *two* of the following :

(*a*) If $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibrations of a string of length l fixed at both ends find the solution with the following conditions : [7]

(*i*) $y(0, t) = 0$

(*ii*) $y(l, t) = 0$ for all t,

(*iii*) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$ for all x and

(*iv*) $y(x, 0) = \frac{3a}{2l}x, \quad 0 \le x \le \frac{2l}{3}$

$= \frac{3a}{l}(l - x), \quad \frac{2l}{3} \le x \le l.$

(*b*) A rod of length l with insulated sides in initially at a uniform temperature x. Both the ends of the rod are kept at zero temperature. Find the temperature at any point and at any time t, use $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. [6]

(*c*) A rectangular plate is bounded by $x = 0$, $x = a$, $y = 0$, $y = b$. Its surfaces are insulated and temperature along three edges $x = 0$, $x = a$, $y = 0$ is maintained at 0°C, while the fourth edge $y = b$ is maintained at constant temperature u_0 until steady state is reached. Find steady state temperature $u(x, y)$. [6]

Or

8. Solve any *two* of the following :

(*a*) If $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibrations of string of length l fixed at both ends, find the solution with boundary conditions : [7]

(*i*) $y(0, t) = 0, \quad \forall \; t$

(*ii*) $y(l, t) = 0, \quad \forall \; t$ and initial conditions

(*iii*) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0, \quad \forall \; x$

(*iv*) $y(x, 0) = k(lx, x^2), \quad 0 \leq x \leq l.$

(*b*) The temperature at any point of a insulated metal rod of one meter length is governed by the differential equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. Find $u(x, t)$ subject to the following conditions : [6]

(*i*) $u(0, t) = 0°C$

(*ii*) $u(1, t) = 0°C$

(*iii*) $u(x, 0) = 50°C$,

and hence find the temperature in the middle of the rod at subsequent time.

(*c*) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which satisfies the conditions : [6]

$u(0, y) = u(l, y) = u(x, 0) = 0$ and

$u(x, a) = \sin \frac{n\pi}{l} x, \quad 0 \le x \le l.$