Total No. of Questions—8] [Total No. of Printed Pages—4+1

Seat No.	

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S.E. (Civil) (First Semester) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. **:—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** and Q. No. **7** *or* Q. No. **8**.

(*ii*) Figures to the right indicate full marks.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $(D^3 - D^2 + 4D - 4)y = e^x$.

(*ii*) $(D^2 + 4)y = \sec 2x$.

(by method of variation of parameters)

(*iii*) $x^3 \dfrac{d^3y}{dx^3} + x^2 \dfrac{d^2y}{dx^2} - 2y = \dfrac{1}{x^3}$.

(*b*) Solve the following equations by using Gauss elimination method : [4]

$$2x_1 + 4x_2 + x_3 = 3$$
$$3x_1 + 2x_2 - 2x_3 = -2$$
$$x_1 - x_2 + x_3 = 6$$

Or

2. (*a*) A light horizontal strut AB of length l is freely pinned at A and B and is under the action of equal and opposite

P.T.O.

compressive forces P at each of its ends and carries a load W at its centre. How that the deflection at its centre is :

$$\frac{W}{2P}\left[\frac{1}{n}\tan\frac{nl}{2}-\frac{l}{2}\right]$$

where $n^2 = \dfrac{P}{EI}$. [4]

(*b*) Use Runge-Kutta method of fourth order to obtain y when $x = 1.1$ for [4]

$$\frac{dy}{dx} = x^2 + y^2;$$

$$y(1) = 1.5, \quad h = 0.1$$

(*c*) Solve the following system by Cholesky's method :

$$4x_1 + 2x_2 + 14x_3 = 14$$
$$2x_1 + 17x_2 - 5x_3 = -101$$
$$14x_1 - 5x_2 + 83x_3 = 155$$

[4]

3. (*a*) Calculate first three moments of the following distribution about the mean : [4]

x	***f***
0	1
1	8
2	28
3	56
4	70
5	56
6	28
7	8
8	1

(*b*) If mean and variance of a binomial distribution are 12 and 3 respectively, find $P(r \geq 1)$. [4]

(*c*) Find the directional derivative of $\phi = x^2 - y^2 - 2z^2$ at the point P(2, –1, 3) in the direction PQ where Q(5, 6, 4). [4]

Or

4. (*a*) Prove the following (any *one*) : [4]

(*i*) $\nabla \cdot (r^3 \bar{r}) = 3r(r^2 + 1)$

(*ii*) $\nabla^2 [\nabla \cdot (r^{-2} \bar{r})] = 2r^{-4}$

(*b*) Prove that :

$$\bar{F} = \frac{1}{r}[r^2 \bar{a} + (\bar{a} \cdot \bar{r})\bar{r}]$$

is irrotational. [4]

(*c*) Obtain correlation coefficient between population density and death rate from the data related to 5 cities. [4]

Population density	**Death rate**
200	12
500	18
400	16
700	21
300	10

5. (*a*) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = (x^2 + y^2)\hat{i} + (x^2 - y^2)\hat{j}$ and C is the curve $y = x^2$ joining (0, 0) and (1, 1). [5]

(*b*) Using Gauss divergence theorem, for the vector function $\bar{F} = (x^3 - yz)i - 2x^2y\hat{j} + 2\hat{k}$ evaluate $\iint_S \bar{F}.\, d\bar{S}$, where S is the surface bounding. The cube $x = 0$, $y = 0$, $z = 0$ and $x = a$, $y = a$ and $z = a$. [4]

(*c*) Evaluate using Stokes' theorem $\int_C \bar{F}.\, d\bar{r}$, where $\bar{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and C is the curve $x^2 + y^2 = 1$, $z = y^2$. [4]

Or

6. (*a*) Show that $\bar{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the work done by the force $\bar{F}$ in moving the object from (1, –2, 1) to (3, 1, 4). [5]

(*b*) Evaluate using Stokes' theorem $\iint_S \nabla \times \bar{F} \,.\, d\bar{S}$, where $\bar{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and $z \geq 0$. [4]

(*c*) Evaluate $\iint_S \bar{r}.\hat{n}\, dS$ over the surface of a sphere of radius 2 with origin as centre. [4]

7. (*a*) Solve $\dfrac{\partial^2 y}{\partial t^2} = c^2 \dfrac{\partial^2 y}{\partial x^2}$ subject to the following conditions : [7]

(*i*) $y(0, t) = 0, \ \forall t$

(*ii*) $y(l, t) = 0, \ \forall t$

(*iii*) $\left(\dfrac{\partial y}{\partial t}\right)_{t=0} = 0$

(*iv*) $y(x, 0) = \frac{3a}{2l}x, 0 < x < \frac{2l}{3}$

$= \frac{3a}{l}(l - x), \frac{2l}{3} < x < l$.

(*b*) An infinitely long plane uniform plate is bounded by two parallel edges in the *y*-direction and an end at right angles to them. The breadth of the plate is π. This end is maintained at the constant temperature 40°C at all points and other edges at zero temperature. Find the steady state temperature $u(x, y)$. [6]

Or

8. (*a*) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the following conditions : [7]

(*i*) u is finite for all t

(*ii*) $u(0, t) = 0, \forall t$

(*iii*) $u(l, t) = 0, \forall t$

(*iv*) $u(x, 0) = \pi x - x^2, 0 \le x \le \pi$.

(*b*) Solve the wave equation : [6]

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

subject to the following conditions :

(*i*) $u(0, t) = 0, \forall t$

(*ii*) $u(\pi, t) = 0, \forall t$

(*iii*) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$

(*iv*) $u(x, 0) = 2x, 0 < x < \pi$.